MATHEMATICS QUIZ

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St.Joseph's College(Autonomous)

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Round 2

Questions

Choose the Correct Answer

$\lim\limits_{x \to 0+} \dfrac{\sin x}{\sqrt{x}}$ is equal to

(a) 1

(b) $\frac{\pi}{2}$

(c) 0

(d) Does not exist

Explanation

Choose the Correct Answer

$\lim_{x \to 0+} \frac{\sin x}{\sqrt{x}}$ is equal to

(a) 1

(b) $\frac{\pi}{2}$

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(a) 1

(b) $\frac{\pi}{2}$

(c) 0

(d) Does not exist

Explanation

Choose the Correct Answer

The limit at $x = 4$ of the function

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases}$$

is equal to

(a) -1

(b) 1

(c) 0

(d) Does not exist

Explanation

Choose the Correct Answer

The limit at $x = 4$ of the function

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases}$$

is equal to

(a) -1

(b) 1

(c) 0

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Explanation

Choose the Correct Answer

The limit at $x = 4$ of the function

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases} \qquad \text{is equal to}$$

(a) -1

(b) 1

(c) 0

(d) Does not exist

Explanation

Choose the Correct Answer

$y = \log_a x$,then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x}\log_a x$

(b) $\frac{1}{x}$

(c) $\frac{1}{x \log_a e}$

(d) $\frac{1}{log_a x}$

Explanation

Choose the Correct Answer

$y = \log_a x$,then $\frac{dy}{dx}$ is equal to

(a) $\frac{1}{x} \log_a x$

(b) $\frac{1}{x}$

(c) $\frac{1}{x \log_a e}$

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Explanation

Choose the Correct Answer

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(a) $\frac{1}{x} \log_a x$

(b) $\frac{1}{x}$

(c) $\frac{1}{x \log_a e}$

(d) $\frac{1}{log_a x}$

Explanation

Choose the Correct Answer

$\lim_{x\to\infty} \left(\frac{x+5}{x-1}\right)^x$ is equal to

(a) 6

(b) e^{-6}

(c) $\frac{1}{6}$

(d) e^6

Choose the Correct Answer

$\lim_{x \to \infty} \left(\frac{x+5}{x-1}\right)^x$ is equal to

(a) 6

(b) e^{-6}

(c) $\frac{1}{6}$

(d) e^{6}

goto exp

Choose the Correct Answer

$\lim_{x\to\infty}\left(\frac{x+5}{x-1}\right)^x$ is equal to

(a) 6

(b) e^{-6}

(c) $\frac{1}{6}$

(d) e^{6}

goto exp

Choose the Correct Answer

$\lim_{x\to1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)=3$, then n is equal to

(a) 2

(b) 3

(c) 0

(d) 4

Choose the Correct Answer

$\lim\limits_{x\to1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)=3$, then n is equal to

(a) 2

(b) 3

(c) 0

(d) 4

goto exp

Choose the Correct Answer

$\lim\limits_{x \to 1} \left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1} \right) = 3$, then n is equal to

(a) 2

(b) 3

(c) 0

(d) 4

goto exp

Choose the Correct Answer

$$\int_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx$$ is equal to

(a) 2

(b) -1

(c) 0

(d) 1

Choose the Correct Answer

$$\int_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx$$ is equal to

(a) 2

(b) -1

(c) 0

(d) 1

goto exp

Choose the Correct Answer

$$\int_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx$$ is equal to

(a) 2

(b) -1

(c) 0

(d) 1

goto exp

Choose the Correct Answer

$\lim_{x \to 0-} \frac{\sin x}{\sqrt{x}}$ is equal to

(a) -1

(b) $\frac{\pi}{2}$

(c) 0

(d) Does not exist

Explanation

Choose the Correct Answer

$\lim\limits_{x \to 0-} \frac{\sin x}{\sqrt{x}}$ is equal to

(a) -1

(b) $\frac{\pi}{2}$

(c) 0

(d) Does not exist

Explanation

Choose the Correct Answer

$\lim_{x \to 0-} \frac{\sin x}{\sqrt{x}}$ is equal to

(a) -1

(b) $\frac{\pi}{2}$

(c) 0

(d) Does not exist

Explanation

Answer

Answer

$$\lim_{x\to 0+} \frac{\sin x}{\sqrt{x}} = \lim_{x\to 0+} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right)$$

Return

Answer

$$\lim_{x \to 0+} \frac{\sin x}{\sqrt{x}} = \lim_{x \to 0+} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right)$$
$$= \lim_{x \to 0+} \left(\frac{\sin x}{x} \right) \lim_{x \to 0+} \sqrt{x}$$

Return

Answer

$$
\begin{aligned}
\lim_{x\to 0+} \frac{\sin x}{\sqrt{x}} &= \lim_{x\to 0+} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right) \\
&= \lim_{x\to 0+} \left(\frac{\sin x}{x} \right) \lim_{x\to 0+} \sqrt{x} \\
&= 1 \times 0
\end{aligned}
$$

Return

Answer

$$\lim_{x\to 0+} \frac{\sin x}{\sqrt{x}} = \lim_{x\to 0+} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right)$$

$$= \lim_{x\to 0+} \left(\frac{\sin x}{x} \right) \lim_{x\to 0+} \sqrt{x}$$

$$= 1 \times 0$$

$$= 0$$

Return

Answer

Answer

$$\lim_{x \to 4+} f(x) = \lim_{x \to 4+} \frac{|x-4|}{x-4}$$

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases}$$

$\therefore$ Limit does not exist

Answer

$$f(x)=\begin{cases}\dfrac{|x-4|}{x-4} & ,x\neq 4\\ 0 & ,x=4\end{cases}$$

$$\begin{aligned}\lim_{x\to 4+} f(x) &= \lim_{x\to 4+}\frac{|x-4|}{x-4}\\ &= \lim_{x\to 4+}\frac{x-4}{x-4} \quad [\because x>4, (x-4)>0]\end{aligned}$$

$\therefore$ Limit does not exist

Answer

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases}$$

$$\begin{aligned} \lim_{x \to 4+} f(x) &= \lim_{x \to 4+} \frac{|x-4|}{x-4} \\ &= \lim_{x \to 4+} \frac{x-4}{x-4} \quad [\because x > 4, (x-4) > 0] \\ &= 1 \end{aligned} \tag{1}$$

$\therefore$ Limit does not exist

Answer

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases}$$

$$\begin{aligned} \lim_{x \to 4+} f(x) &= \lim_{x \to 4+} \frac{|x-4|}{x-4} \\ &= \lim_{x \to 4+} \frac{x-4}{x-4} \quad [\because x > 4, (x-4) > 0] \\ &= 1 \end{aligned} \tag{1}$$

$$\lim_{x \to 4-} f(x) = \lim_{x \to 4-} \frac{-(x-4)}{x-4} \quad [\because x < 4, (x-4) < 0]$$

$\therefore$ Limit does not exist

Return

Answer

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases}$$

$$\begin{aligned} \lim_{x \to 4+} f(x) &= \lim_{x \to 4+} \frac{|x-4|}{x-4} \\ &= \lim_{x \to 4+} \frac{x-4}{x-4} \quad [\because x > 4, (x-4) > 0] \\ &= 1 \qquad (1) \end{aligned}$$

$$\begin{aligned} \lim_{x \to 4-} f(x) &= \lim_{x \to 4-} \frac{-(x-4)}{x-4} \quad [\because x < 4, (x-4) < 0] \\ &= -1 \qquad (2) \end{aligned}$$

$\therefore$ Limit does not exist

Return

Answer

$$f(x) = \begin{cases} \dfrac{|x-4|}{x-4} & , x \neq 4 \\ 0 & , x = 4 \end{cases}$$

$$\lim_{x \to 4+} f(x) = \lim_{x \to 4+} \frac{|x-4|}{x-4}$$

$$= \lim_{x \to 4+} \frac{x-4}{x-4} \quad [\because x > 4, (x-4) > 0]$$

$$= 1 \qquad (1)$$

$$\lim_{x \to 4-} f(x) = \lim_{x \to 4-} \frac{-(x-4)}{x-4} \quad [\because x < 4, (x-4) < 0]$$

$$= -1 \qquad (2)$$

$$\therefore \lim_{x \to 4+} f(x) \neq \lim_{x \to 4-} f(x)$$

$\therefore$ Limit does not exist

Return

Answer

Answer

$$y = \log_a x$$

Return

Answer

$$y = \log_a x = \frac{\log_e x}{\log_e a}$$

Return

Answer

$$
\begin{aligned}
y &= \log_a x \\
&= \frac{\log_e x}{\log_e a} \\
\Longrightarrow \frac{dy}{dx} &= \frac{1}{\log_e a}\frac{1}{x}
\end{aligned}
$$

Return

Answer

Answer

$$\lim_{x\to\infty}\left(\frac{x+5}{x-1}\right)^x = \lim_{x\to\infty}\left(\frac{1+\frac{5}{x}}{1-\frac{1}{x}}\right)^x$$

Answer

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Answer

$$\lim_{x\to\infty}\left(\frac{x+5}{x-1}\right)^x = \lim_{x\to\infty}\left(\frac{1+\frac{5}{x}}{1-\frac{1}{x}}\right)^x$$

$$= \frac{\lim_{x\to\infty}\left(1+\frac{5}{x}\right)^x}{\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^x}$$

$$= \frac{e^5}{e^{-1}}$$

Answer

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$$= \frac{e^5}{e^{-1}}$$

$$= e^6$$

Return

Answer

Answer

$$\lim_{x \to 1} \left(\frac{x + x^2 + x^3 + \cdots + x^n - n}{x - 1} \right)$$

Answer

$$\lim_{x\to 1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)$$
$$=\lim_{x\to 1}\left(\frac{1+2x+3x^2+\cdots+nx^{n-1}}{1}\right)\quad [L'Hospitol's \quad rul$$

Answer

$$\lim_{x\to 1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)$$

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$$=1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

Answer

$$\lim_{x \to 1} \left(\frac{x + x^2 + x^3 + \cdots + x^n - n}{x - 1} \right)$$

$$= \lim_{x \to 1} \left(\frac{1 + 2x + 3x^2 + \cdots + nx^{n-1}}{1} \right) \quad [L'Hospitol's \quad rul$$

$$= 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$$\lim_{x \to 1} \left(\frac{x + x^2 + x^3 + \cdots + x^n - n}{x - 1} \right) = 3$$

Answer

$$\lim_{x\to 1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)$$

$$=\lim_{x\to 1}\left(\frac{1+2x+3x^2+\cdots+nx^{n-1}}{1}\right)\quad [L'Hospitol's \quad rul$$

$$=1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

$$\lim_{x\to 1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)=3$$

$$\implies \frac{n(n+1)}{2}=3$$

Answer

$$\lim_{x\to1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)$$

$$=\lim_{x\to1}\left(\frac{1+2x+3x^2+\cdots+nx^{n-1}}{1}\right) \quad [L'Hospitol's \quad rul$$

$$=1+2+3+\cdots+n=\frac{n(n+1)}{2}$$

$$\lim_{x\to1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)=3$$

$$\Longrightarrow \frac{n(n+1)}{2}=3$$

$$\Longrightarrow n=2$$

Answer

$$\lim_{x\to 1}\left(\frac{x+x^2+x^3+\cdots+x^n-n}{x-1}\right)$$

$$=\lim_{x\to 1}\left(\frac{1+2x+3x^2+\cdots+nx^{n-1}}{1}\right) \quad [L'Hospitol's \quad rul$$

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Answer

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$$\implies n=2$$

Return

Answer

Answer

We know that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Return

Answer

We know that $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

Here $a+b=0$

Return

Answer

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Here $a+b=0$

$$\implies \int\limits_{-1}^{1} f(x)dx = \int\limits_{-1}^{1} f(-x)dx$$

Return

Answer

We know that $\int\limits_a^b f(x)dx = \int\limits_a^b f(a+b-x)dx$

Here $a+b=0$

$$\implies \int\limits_{-1}^{1} f(x)dx = \int\limits_{-1}^{1} f(-x)dx$$

$$\implies \int\limits_{-1}^{1} \sqrt{1+x+x^2}dx = \int\limits_{-1}^{1} \sqrt{1-x+x^2}dx$$

Return

Answer

We know that $\int\limits_a^b f(x)dx = \int\limits_a^b f(a+b-x)dx$

Here $a+b=0$

$$\implies \int\limits_{-1}^{1} f(x)dx = \int\limits_{-1}^{1} f(-x)dx$$

$$\implies \int\limits_{-1}^{1} \sqrt{1+x+x^2}dx = \int\limits_{-1}^{1} \sqrt{1-x+x^2}dx$$

$$\implies \int\limits_{-1}^{1}\left(\sqrt{1+x+x^2}-\sqrt{1-x+x^2}\right)dx = 0$$

Return

Answer

Answer

$$\lim_{x\to 0-} \frac{\sin x}{\sqrt{x}} = \lim_{x\to 0-} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right)$$

Answer

$$\lim_{x \to 0-} \frac{\sin x}{\sqrt{x}} = \lim_{x \to 0-} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right)$$

$$= \lim_{x \to 0-} \left(\frac{\sin x}{x} \right) \lim_{x \to 0-} \sqrt{x}$$

Return

Answer

$$\lim_{x\to 0-} \frac{\sin x}{\sqrt{x}} = \lim_{x\to 0-} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right)$$
$$= \lim_{x\to 0-} \left(\frac{\sin x}{x} \right) \lim_{x\to 0-} \sqrt{x}$$

$$x \to 0- \implies x < 0$$

Return

Answer

$$\lim_{x\to 0-} \frac{\sin x}{\sqrt{x}} = \lim_{x\to 0-} \left(\frac{\sin x}{x} \frac{x}{\sqrt{x}} \right)$$
$$= \lim_{x\to 0-} \left(\frac{\sin x}{x} \right) \lim_{x\to 0-} \sqrt{x}$$

$$x \to 0- \implies x < 0$$
$$\implies \lim_{x\to 0-} \sqrt{x} \quad \text{Does not exist}$$

Return